High-Speed Balancing of Rotors with Overhangs: When is Overhang Likely to Cause Problems?

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Abstract

In high-speed balancing of flexible rotors with overhangs, it is important to determine ahead of time if the overhang needs to be restrained while the rotor is being tested in the bunker. This is done by adding a stub shaft to the rotor and a third pedestal to the overall system. Since the bearing properties are rarely known precisely and have a considerable influence on the rotor's dynamics, the model complexity increases tremendously if a third pedestal is needed during the balancing operation. From a balance engineer's viewpoint, it is therefore important to know ahead of time, without modeling the entire rotor, if the overhangs are likely to cause a problem during the balancing operation. This paper presents a criterion for identifying rotors with shaft overhangs that are likely to exhibit dangerous behavior while balancing. The proposed approach is based on determining the influence coefficients of the rotor overhang, and provides quick estimates of the L-mode frequency of the overhang. Two additional criteria based on further simplifications of the influence coefficient approach are also presented. Numerical results from four industrial rotors recently balanced indicate that the proposed methods are effective in determining the need for a third pedestal without having to resort to extensive rotor modeling.

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Introduction

A flexible rotor, such as a generator rotor, typically operates above its first or second natural frequency (critical speed). During the start up and shut down operations, the rotor will pass through these critical speeds and will encounter the problems related to resonance. In general, the response of the rotor during these transient periods is well damped by the bearings and does not present any major problems. Nevertheless there is still concern for certain kinds of rotors, the ones with long overhangs, because the vibration levels in the overhang may permanently damage the rotor by bending the overhang during these transients. The situation is aggravated during the normal balancing procedure due to the following two facts (i) the rotor is uncoupled from the turbine softening the overhang rigidity, and (ii) it is set in softer balance pedestals that shift the first few natural frequencies towards, and into the normal operating speed range.

The first vibration mode of the overhang portion of the rotor resembles an "L" shape (see Fig. 1), and is referred to as the "L-mode" herein. Rotors with long overhangs present challenges to the balancer who needs to know in advance (before a complete model of the rotor is created) if the rotor is likely to experience an L-mode. L-modes can be prevented during balancing by adding a stub shaft and a third bearing to the system to change its natural mode shape and increase the damping in the system to some extent. However, the addition of a third pedestal and the accompanying modifications needed for balancing make the balancing operation much more time consuming (by a factor of three or more) and therefore more costly.

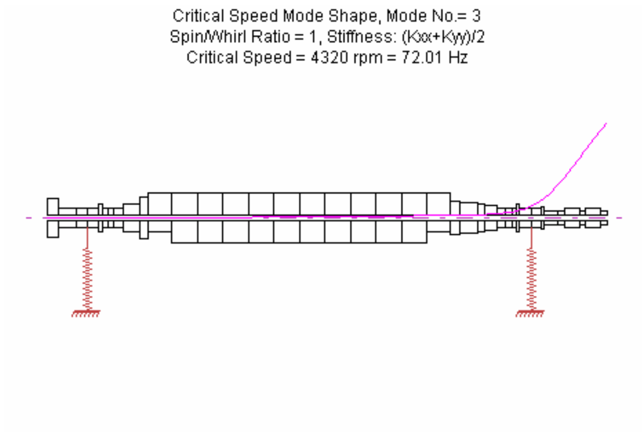

Fig. 1. Rotor with overhang in L-mode.

It is possible to numerically model the rotor to fully understand its behavior in cases where the balancer suspects an L-mode may occur. Complete rotor modeling is time consuming and expensive. On average, an industrial rotor takes about three to four hours to model, with an additional 6-8 hours (twice the modeling time) needed to fine-tune the model to match the response of the real rotor with that of the computer model. This is due to large nonlinearities in the bearing behavior as well as uncertainties in pedestal properties. For reasons of expediency, it is therefore desirable to have a discrimination criterion that can help separate quickly "dangerous" cases from those that are not, without the need for complete rotor modeling. This helps the balance engineer decide which rotors need to have their overhangs restrained during the high-speed balance operations and which rotors don't need it.

There are some rules-of-thumb, currently in use in the industry, wherein the gravity sag of the overhang and the need for a third pedestal are related. These rules are largely based on empirical evidence and lack rigorous theoretical justification. Furthermore, even though rotor dynamic modeling has received considerable attention in the literature, nothing is available which can help a balancer discern quickly, without having to resort to complete rotor modeling, if the overhang is likely to cause problems during the balancing operation.

Today, most of the existing methods for rotor dynamics modeling allow for the inclusion of overhangs as an integral part of a rotor model. Based on the transfer matrix approach developed by Myklestad and Prohl, Lund developed a transfer matrix method in the late 70's [1], [2] which allows for the inclusion of overhangs in the model. A similar approach that includes gyroscopic effects for shafts with overhangs is described by Rao [3]. Many other works based on the transfer matrix methods can be found in the literature [4].

Since the development of finite element (FE) techniques, during the last 25 years considerable progress has been made in developing refined rotor models that include gyroscopic effects, different mass elements, disk elements, support and bearing characteristics [5]. A good summary of these developments can be found in Ehrich [6]. A dynamic element method has recently been presented by Hong and Park [7] for providing exact solutions for multi-stepped rotor-bearing systems. For analyzing the transient dynamics of rotary systems with unbalance, Huang and Wang [8] have developed a numerical method that combines the FE method, the transfer matrix method and numerical integrations. A frequent problem with the use of FE modeling is that the models frequently are not general enough to capture non-linearities due to bearing characteristics. Furthermore, the uncertainty in the pedestal parameters is usually so large so that a great deal of model fine-tuning and expertise is expected from the analyst in order to match the model results with those obtained from the actual rotor.

In addition to the numerical methods discussed above, an analytical solution has recently been presented by Murphy [9] for predicting lateral vibration in beams with overhangs. However, this work is restricted to beams with symmetric overhangs and constant moment of inertia throughout the length, and is this of limited practical value. As can been seen from the reviewed literature, to our knowledge, there is nothing available in the literature that specifically address balancing of rotors with long overhangs and/or presents a threshold criterion to distinguish dangerous cases without resorting to complete rotor modeling.

To overcome this difficulty, this work presents an analytical explanation of the overhang problem and a simplified method of calculation of the overhang critical speed. The main advantage of this approach is that avoids all the major sources of nonlinearity and un-modeled dynamics of the previous approaches, bearings and pedestals, but yet retains the essence of the rotor dynamic behavior. An additional advantage of the proposed approach is its ease of implementation, and it provides a conservative result which can be potentially very useful in industrial settings.

Vibrations in the Overhang

The L-mode of vibration occurs when the difference between the rotor stiffness to mass relation and the overhang stiffness to mass relation is large enough to cause the overhang to act (by itself) as a cantilever beam. Under these conditions, the natural frequency of the rotor for an L-mode approaches the first natural frequency of the overhang when the overhang itself is analyzed as a cantilevered beam.

The Rayleigh's quotient [10] for a rotor is defined as

$$R(u) = \frac{\{u\}^T [K] \{u\}}{\{u\}^T [M] \{u\}} \tag{1}$$

where {u} is an arbitrarily selected displacement vector. Since the Rayleigh's coefficient approaches the critical frequency ω_1 from above, $R(u) \geq \omega_1^2$. For $u = u_1$, the first mode shape, $R(u_1) = \omega_1^2$.

If the displacement vector {u} is partitioned as $\{\{u_{oh}\},\{u_{body}\}\}$, then for the L-mode, the displacement components corresponding to the overhang portion will be the most significant ones in terms of computing the Rayleigh quotient. This is because the components of the overhang (u_{oh}) are the ones that have the bigger displacements compared with the rest of the rotor (u_{body}).

$$R(u) = \frac{\left\{\{u_{oh}\}\{u_{body}\}\right\}^T [K] \left\{\{u_{oh}\}\{u_{body}\}\right\}}{\left\{\{u_{oh}\}\{u_{body}\}\right\}^T [M] \left\{\{u_{oh}\}\{u_{body}\}\right\}} \tag{2}$$

Here,

$$K = \begin{bmatrix} \boxed{\begin{matrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1+k_2) & -k_2 \\ 0 & -k_2 & \ddots \end{matrix}}_{oh} & 0 \\ 0 & \boxed{\begin{matrix} k_m + k_{m+1} & \ddots & 0 \\ \ddots & \ddots & -k_{n-1} \\ 0 & -k_{n-1} & k_n \end{matrix}}_{body} \end{bmatrix} \qquad u = \begin{Bmatrix} \begin{Bmatrix} u_1 \\ \vdots \\ u_{m-1} \end{Bmatrix}_{oh} \\ \begin{Bmatrix} u_m \\ \vdots \\ u_n \end{Bmatrix}_{body} \end{Bmatrix} \tag{3}$$

It follows that as the L-mode becomes more and more sharp in its shape, then the natural frequency of the mode approaches the first natural frequency of the overhang. For an L-mode, in the limit, we get:

$$R(u) = \frac{\left\{\{u_{oh}\}\{u_{body}\}\right\}^T [K] \left\{\{u_{oh}\}\{u_{body}\}\right\}}{\left\{\{u_{oh}\}\{u_{body}\}\right\}^T [M] \left\{\{u_{oh}\}\{u_{body}\}\right\}} \approx \left. \frac{\{u_{oh}\}^T [K_{oh}] \{u_{oh}\}}{\{u_{oh}\}^T [M_{oh}] \{u_{oh}\}} \right|_{n=1} \approx \omega_{1oh}^2 \tag{4}$$

This is an important result because it allows us to establish a threshold criterion to discriminate between differing rotors, namely: "If the overhang exhibits its first natural frequency within the operating speed of the rotor, then it is necessary to model the rotor to fully understand its behavior and a use of third pedestal is recommended to prevent potential damage."

The problem that we encounter with this approach is that while it establishes a threshold criterion, it does not help us from a computational point of view because it is necessary to know priory the first mode shape {u} for the overhang. Furthermore, it is not practical for implementation unless a computer code is available. On the other hand, the main advantage of this approach is that by modeling only the overhang as a cantilevered beam, we avoid modeling bearings and supports. These are the main sources of uncertainty and nonlinearity in an FE model. A second advantage of this approach is that since the dynamical model is considerable smaller (because only the overhang is modeled instead of the complete rotor), the model is much quicker to build and run.

For above-mentioned reasons, it is therefore desirable to find a procedure that allows for the computation of the first natural frequency of the overhang in an easy manner. The Dunkerley's formulation utilizing influence coefficients [11] is an alternate approach to approximate the first natural frequency of a rotor. Unlike Rayleigh's quotients that approach the natural frequency from above, Dunkerley's method approximates the fundamental frequency from below. This fact will compensate, to some extent, the error introduced by approximating the rotor behavior using Rayleigh's formulation. On the other hand, since Dunkerley's provides a lower bound for the natural frequency, the results will be on the conservative side for a balance engineer.

Approximation of the First Natural Frequency of Overhangs

The Dunkerley's method provides a quick estimate of the first natural frequency of the overhang because the influence coefficients of the overhang can be directly determined experimentally from rotor measurements, and hence can provide very reliable information. The procedure is summarized as follows.

By measuring the deflection δ_i of the rotor's overhang at the i^{th} location under a known load f_j, the influence coefficient a_{ij} is known. The flexibility matrix $A = [a_{ij}] = K^{-1}$. The eigenvalue problem for the rotor system is now given as:

$$\begin{aligned} [M]\ddot{X} + [K]X &= 0 \\ \left|[K]^{-1}[M] + \lambda[I]\right| &= 0 \\ \left|[A][M] + \lambda[I]\right| &= 0 \end{aligned} \tag{5}$$

Since the trace of a matrix is equal to the sum of its eigenvalues,

$$tr\left([A][M]\right) = \sum_{i=1}^{n} a_{ii} m_{ii} = \sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \tag{6}$$

Then, if $\frac{1}{\omega_1} \gg \frac{1}{\omega_2} > \ldots > \frac{1}{\omega_n}$; which for the case of a rotor's overhang is reasonably true (usually the second natural frequency is at least 5 to 6 times bigger), we get

$$\omega_1 \cong \frac{1}{\sqrt{\sum_{i=1}^{n} a_{ii} m_{ii}}} \tag{7}$$

Eq. (6) is very easy to compute using a calculator or a spreadsheet. Furthermore the coefficients a_{ii} can be easily measured experimentally or computed from the stiffness matrix K. The values m_{ii} (diagonal mass matrix elements) are known from the dimensions of the rotor.

Presented next is a procedure to evaluate the influence coefficients so that ω_1 can be expressed in terms of diameters and lengths for each overhang element. For overhang approximated using beam elements (Fig. 2), the influence coefficient to at the first station is given as:

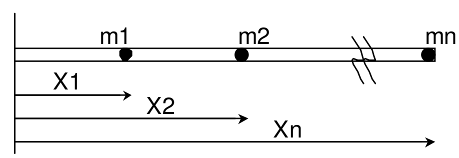

Fig. 2. Overhang approximated as beam elements.

$$a_{1,1} = \frac{x_1^3}{3EI_1} \tag{8}$$

For successive influence coefficients $a_{2,2}$ $a_{3,3}$ …$a_{i,i}$ we first establish a relationship among them. Let $\beta_{i,j}$ denote the quotient between two adjacent influence coefficients defined as follows:

$$\beta_{i,j} = \frac{a_{i,i}}{a_{j,j}} \rightarrow a_{2,2} = \beta_{1,2} a_{1,1} \tag{9}$$

Finally the relationship between any two coefficients will be product of their associate beta coefficients:

$$\begin{aligned} \beta_{1,5} &= \beta_{1,2}\beta_{2,3}\beta_{3,4}\beta_{4,5} \\ \beta_{i,j} &= \beta_{i,2}\beta_{i+1,3}\ldots\beta_{j-1,j} = \prod_{k=i}^{j-1} \beta_{k,k+1} \end{aligned} \tag{10}$$

See the appendix for a derivation of beta coefficients

And in general each influence coefficient will be

$$\beta_{j-1,j} = \frac{\left[\dfrac{\left(x_j - x_{j-1}\right)^3}{3I_j} + \dfrac{x_{j-1}^2(x_{j-2} - x_j) + (x_j^2 - x_{j-2}^2)x_{j-1}}{I_{j-1}}\right]}{Ea_{j-1,j-1}} + 1$$

$$a_{j,j} = a_{j-1,j-1}\beta_{j-1,j} \qquad (11)$$

$$m_j = \rho\frac{\pi d_j^2\left(x_j - x_{j-1}\right)}{4}$$

The estimate of the natural frequency (in rpm) finally is:

$$\omega_1 \approx \frac{30}{\pi\sqrt{\sum_{j=1}^{n} a_{j,j} m_j}} \qquad \text{[Expressed in rpm]} \qquad (12)$$

Discussion and Practical Results:

The Dunkerley's approach works very well for cases where the fundamental frequency is well below other natural frequencies. On the generator rotors tested, the second frequency of the overhang portion was seen to be located anywhere from 6 to 15 times the fundamental frequency[1]. Furthermore, a small error (of the order of 10 %) was encountered between the L-mode frequencies obtained using FE modeling of the entire rotor versus those obtained by the modeling of overhang only.

Because of these two favorable observations, further simplifications are possible which yield simplified expressions and threshold values that can be used to quickly identify "dangerous" cases that are likely to cause problems during balancing. Two criteria that are investigated herein include: a) determining the gravity sag of the rotor tip, and b) analyzing the L/D relationship over the rotor overhang.

a) The rotor overhang can be seen as a one-member cantilever beam with all its mass located at the free end of the beam. In this case, the influence coefficient is given simply as $a = sag/mg$. The natural frequency $\omega = \sqrt{g/sag}$ is now given (in rpm) as:

$$\omega = \frac{30}{\pi\sqrt{\frac{sag}{g}}} \tag{13}$$

This expression establishes a relationship between the natural frequency and the sag. In our case, since the threshold frequency is 4000 rpm, the critical value of sag is 0.0022 inches. Therefore, rotors with a static tip deflection (sag) in the overhang in excess of 0.0022 inches will require a third pedestal during the balancing operation.

b) Along a similar line of thinking, instead of measuring maximum sag at the rotor tip, we can calculate the influence coefficient based on a distributed load cantilevered beam formulation. For a uniformly distributed load, the tip deflection in terms of the dimensionless parameter $\alpha = L/D$ of the overhang is

$$sag = \frac{wL^4}{8EI} = \frac{\rho AgL^4}{8EI} = 2\frac{\rho gD^2\alpha^4}{E} \tag{14}$$

Here w denotes the load per unit length, A is the CS area, and I is the moment of inertia of the overhang. The fundamental frequency of the overhang (in rpm) is now given as

$$\omega_1 = \frac{30}{\pi\alpha^2 D\sqrt{\frac{2\rho}{E}}} \tag{15}$$

This expression gives a threshold value of α as a function of the overhang diameter D. For a natural frequency of 4000 rpm, Table 1 lists threshold values of α for different values of D. Thus to avoid an L-mode and the need for a third pedestal, the

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[1] For a cantilevered beam, the second frequency is approximately 6.25 times the fundamental frequency.

L/D of the overhang portion of the rotor should be smaller than the critical values of α given in Table 1.

Since the rotor diameter varies along its length, the biggest problem we encounter with this approach is which value of D should be used to calculate the ratio L/D. From experience it has been seen that the best results are obtained by choosing D as the smaller diameter closest to the retaining ring. The distance L is measured from the main stiffness change compared with the body stiffness; usually (but not necessarily) this occurs at the end of the retaining rings. It is seen that setting $\alpha = 4$ yields a conservative threshold in all cases.

Comparing results

Four generator rotors sent for balancing to ReGENco by utility companies are chosen to illustrate the effectiveness and weakness of the proposed methods. These rotors underwent a high-speed balance at 3600 rpm with an over-speed test conducted at 3960 rpm for 1 minute. Numerical results from finite element (DyRoBes) simulation of the entire rotor, FE modeling of the overhang alone, as well as the influence coefficient approach, and the simplified gravity sag and L/D approaches are shown in Table 2. This table also includes experimentally observed critical speed for the L-mode also.

Based on the results presented in Table 2, it is clear that by just modeling the overhang alone, we can distinguish between rotors which will require a 3rd pedestal with good confidence. The maximum error in L-mode frequency encountered between FE models of the entire rotor and the FE models of the overhang only is 13%. When the results of the overhang modeling are compared with actual critical speed observed during the test, the maximum deviation drops to 5%. This improvement in agreement is largely due to the fact that the biggest sources of model errors are uncertainties in the bearing properties and pedestal nonlinearities. These errors are eliminated in overhang alone models. In this way we save time and effort avoiding most of the problems encountered during FEs analysis as well as eliminating the time spent fine-tuning the FE model.

For most cases analyzed herein, the influence coefficient approach yields results which are within 10% of experimentally observed critical speed except for one case with rotor 2 where there is a 22% discrepancy. The results from the two simplified approaches, namely the gravity sag method and the L/D approach also compare favorably with those obtained using the influence coefficient method. It is clear that both of the simplified methods also provide a quick discrimination criterion which are likely to be very useful to a balance engineer.

Conclusions

This paper has presented simplified criteria for quickly identifying rotors with long-overhang that are likely to exhibit “dangerous” behavior during high-speed balancing. An approximate method of computing rotor’s L-mode natural frequency is presented and a numerical comparison shows its effectiveness and limitations. In addition, an explanation of why long-overhang rotors show an independent behavior of its overhang while balancing has also been presented.

The present approach is not intended to replace FE analysis and modeling, instead it is intended to provide a quick approach to help distinguish non-dangerous cases from the dangerous ones. It is clear that in cases flagged as likely to exhibit “dangerous” vibrations in the overhang, further detailed analysis needs to be done in order to make a decision of whether or not a third bearing is truly necessary. Since the influence coefficients, gravity sag, and the L/D relationship can be (experimentally) measured quickly and in a reliable manner, these approaches are expected to be beneficial to the field engineer interested in evaluating vibrations in the overhangs.

Acknowledgments

The authors acknowledge several fruitful discussions on this work with Zlatan Rasic from Z-R consulting and Joe Rauter at ReGENco, LLC. Financial support of this research from ReGENco, LLC through a graduate internship to the lead author is also gratefully acknowledged. Experimental data on various rotors reported herein was obtained from high-speed balance and overspeed bunker at ReGENco. The authors are also grateful to Prof. Gunter for providing them with a copy of the academic version of the DyRoBeS program.

Appendix

Using Mohr's approach to compute the deflection of a cantilever beam at the load point due to a concentrated load acting at the same location.

$$y_{j,j} = P\left[\frac{(x_j - x_{j-1})^3}{3EI_j} + \sum_{i=j-1}^{2} \frac{(x_j - x_i)^2 (x_i - x_{i-1}) + (x_j - x_i)\dfrac{(x_i - x_{i-1})^2}{2}}{EI_i} + \sum_{i=j-1}^{2}\left[\frac{(x_i - x_{i-1})^2 (x_j - x_i)}{2EI_i} + \frac{(x_i - x_{i-1})^3}{3EI_i}\right]\right]$$

$$y_{j,j} = \frac{P}{E}\left[\frac{(x_j - x_{j-1})^3}{3I_j} + \sum_{i=j-1}^{2} \underbrace{\frac{(x_j - x_i)^2 (x_i - x_{i-1})}{I_i} + \frac{(x_j - x_i)(x_i - x_{i-1})^2}{I_i} + \frac{(x_i - x_{i-1})^3}{3I_i}}_{C}\right] \quad (16)$$

Calling C to the expression inside the summation and expanding the summation

$$a_{j,j}=\frac{1}{E}\left[\frac{(x_j-x_{j-1})^3}{3I_j}+C\big|_{i=j-1}+\sum_{i=j-2}^{2}C\right]$$

$$\beta_{j-1,j}=\frac{a_{j,j}}{a_{j-1,j-1}}=\frac{\frac{1}{E}\left[\frac{(x_j-x_{j-1})^3}{3I_j}+C\big|_{i=j-1}+\sum_{i=j-2}^{2}C\right]}{\frac{1}{E}\left[\frac{(x_{j-1}-x_{j-2})^3}{3I_{j-1}}+\sum_{i=j-2}^{2}C\right]}=\frac{\left[\frac{(x_j-x_{j-1})^3}{3I_j}+C\big|_{i=j-1}\right]}{\left[\frac{(x_{j-1}-x_{j-2})^3}{3I_{j-1}}+\sum_{i=j-2}^{2}C\right]}-\frac{\frac{(x_{j-1}-x_{j-2})^3}{3I_{j-1}}}{\left[\frac{(x_{j-1}-x_{j-2})^3}{3I_{j-1}}+\sum_{i=j-2}^{2}C\right]}+1$$

$$\beta_{j-1,j}=\frac{\left[\frac{(x_j-x_{j-1})^3}{3I_j}+C\big|_{i=j-1}-\frac{(x_{j-1}-x_{j-2})^3}{3I_{j-1}}\right]}{\left[\frac{(x_{j-1}-x_{j-2})^3}{3I_{j-1}}+\sum_{i=j-2}^{2}C\right]}+1 \tag{17}$$

Evaluating C at i=j-1 and grouping common factors

$$\beta_{j-1,j}=\frac{\left[\frac{\left(x_j-x_{j-1}\right)^3}{3I_j}+\frac{x_{j-1}^2(x_{j-2}-x_j)+(x_j^2-x_{j-2}^2)x_{j-1}}{I_{j-1}}\right]}{Ea_{j-1,j-1}}+1 \tag{18}$$

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Table 1. Critical alpha values as a function of rotor diameter.

D (in)	10	12	14	16	18	20
Alpha	5.79458	5.2897	4.89731	4.58102	4.31902	4.09738

Table 2. Results from rotor modeling using FE and proposed approaches.

Rotor Number	Critical Speed[2]	Complete Rotor Modeling	Required time[3]	Overhang modeling only	Required time	Influence coefficient Approach[4]	Gravity Sag Method[5] (rpm/sag)	Distributed Mass Method[5] (rpm/alpha) *	Was a third pedestal used?
Rotor 1	3820	4320	4 hrs modeling 8 hrs tuning	3757	0.5 hr modeling No tuning	3385	3004 / 0.0039	3580 / 5	Yes
Rotor 2	3040	3163	4 hrs modeling 6 hrs tuning	3032	0.5 hr modeling No tuning	2360	2424 / 0.006	2490 / 6	Yes
Rotor 3 (turbine end)	3860	4058	4 hrs modeling 4 hrs tuning	3920	0.5 hr modeling No tuning	3546	3369 / 0.0031	3580 / 5	No, but close monitoring needed for over speed.
Rotor 3 (excitor end)	Beyond over speed[6]	5826		6590	0.5 hr modeling No tuning	5996	5080 / 0.0013	5600 / 4	No
Rotor 4 (turbine end)	3920	4064	4 hrs modeling 4 hrs tuning	3720	0.5 hr modeling No tuning	3395	3126 / 0.0036	3160 / 5	No, but close monitoring needed for over speed.
Rotor 4 (excitor end)	Beyond over speed	7968		7365	0.5 hr modeling No tuning	5974	5762 / 0.00106	6450 / 3.5	No

[2] Experimentally observed in the balance bunker.
[3] Total time required is modeling time plus tuning time which varies between 4 to 8 hrs depending on the model nonlinearity.
[4] The time required is approximately 15 minutes.
[5] The time required is approximately 5 minutes.
[6] Test not done as speed in excess of 3960 rpm.